THEORETICAL STUDY OF RADIANT HEAT EXCHANGE FOR NON-GRAY NON-DIFFUSE SURFACES IN A SPACE ENVIRONMENT

NASA Research Grant: No. NGR-14-005-036
University of Illinois Budget Code: 46-22-40-356

Semi-Annual Status Report Number 1

Period Covered: February 1965 to August 1965

Prepared by: R. G. Hering
A. F. Houchens
T. Smith

Submitted by: R. G. Hering
Associate Professor of
Mechanical Engineering
266 Mechanical Engineering Bldg.
University of Illinois
Urbana, Illinois
Phone: Area Code 217, 333-0366

Date: August 1, 1965

N65-33275 (ACCESSION NUMBER)
25 (PAGES)
CR 64669 (NASA CR OR TMX OR AD NUMBER)
1 (THRU)
(CODE)
33 (CATEGORY)
FACILITY FORM 602

GPO PRICE $
CSFTI PRICE(S) $
Hard copy (HC) 1.00
Microfiche (MF) .50
ff 653 July 65

TABLE OF CONTENTS

		Page
	NOMENCLATURE	1
1.	OBJECTIVES AND SCOPE	3
2.	CURRENT STATUS	4
	2.1 Radiant Heat Exchange for Non-Gray, Non-Diffuse Surfaces in a Space Environment	4
	2.2 Radiation Properties for Rough Surfaces of Conducting Materials	8
	2.3 Adjoint Plate System Analysis	10
	2.4 Radiation Properties for Optically Smooth Plane Surfaces	11
3.	PROPOSED FUTURE RESEARCH	13
4.	REFERENCES	14
5.	FIGURES & TABLES	15
6.	EXPENDITURES	24

NOMENCLATURE

dA_P, dA_Q	=	differential areas at P and Q respectively
a	=	parameter related to rms slope of roughness profile
$E(P)$	=	emissive power at P
$G(R,\vec{i})$	=	collimated solar radiation leaving R in $\vec{i}$ direction per unit time and per unit irradiated area
$G\gamma$	=	kernel in adjoint plate analysis
$I(P,\vec{s})$	=	radiant energy leaving P in $\vec{s}$ direction per unit time, solid angle, and area normal to $\vec{s}$ direction
$K(P,Q)$	=	geometric function $(\cos\theta_P \cos\theta_Q / \pi r_{PQ}^2)$
L	=	length
n	=	refractive index
$\vec{n}_P, \vec{n}_Q$	=	outer surface normals to dA_P and dA_Q respectively
$q''(P)$	=	net radiant energy transfer to P per unit area and time
$R(P)$	=	radiosity at P
r_{PQ}	=	distance between P and Q
$\vec{r},\vec{s},\vec{t},\vec{i},\vec{j}$	=	direction vectors
$\alpha(P,\vec{r})$	=	directional absorptance of surface at P for energy incident in $\vec{r}$ direction
γ	=	angle or rotation in infinite plate geometry
$\epsilon(P,\vec{s})$	=	directional emittance of surface at P in $\vec{s}$ direction
$\epsilon(\theta)$	=	directional emittance predicted by electromagnetic theory for a smooth, plane surface at angle θ from surface normal

ξ = dimensionless distance

η = dimensionless distance

θ = polar angle of $\vec{s}$ and $\vec{j}$ with respect to surface normal

θ_P, θ_Q = angles between line joining P and Q and the respective surface normals

$\rho(P,\vec{r},\vec{s})$ = bi-directional reflectance of surface at P

$\rho(P,\vec{r})$ = directional reflectance of surface at P for radiation incident from $\vec{r}$ direction

$\rho(\theta)$ = reflectance predicted by electromagnetic theory for a smooth plane surface irradiated at angle θ from surface normal

σ = rms height of roughness elements

φ = azimuthal angle of reflected ray with plane of incidence as $\varphi = \pi$

Ψ = polar angle of incident ray measured from reflecting surface normal

$d\omega$ = element of solid angle

Subscripts:

b = black

H = hemispherical

i = incident

L = lower

N = normal

r = reflected

s = solar

sp = specular

U = upper

λ = monochromatic quantity (spectrum between λ and $\lambda + d\lambda$)

1. OBJECTIVES AND SCOPE

Analysis is in progress to assess the importance of such real surface radiating characteristics as non-diffuseness and non-grayness on the radiant heat transfer of surfaces in a space environment. This report outlines two theoretical analyses and two subsidiary investigations necessary to implement these. Section 2.1 presents the basic formulation of the non-gray non-diffuse radiant heat transfer in a space environment for a selected geometry [1]*. Section 2.2 presents a discussion and some observations concerning the bi-directional reflectance model required for the real surface analysis. In Section 2.3, an analysis is outlined which accounts for the directional dependence of emission and reflection for smooth plane surfaces. Finally, results for radiation properties of smooth plane surfaces are presented in Section 2.4.

*Numbers in brackets [ ] designate references in Section 4.

2. CURRENT STATUS

2.1 Radiant Heat Exchange for Non-Gray, Non-Diffuse Surfaces in a Space Environment

The system selected for study [1] of the influence of non-gray, non-diffuse surface properties on radiant heat exchange is two equal length plates of infinite extent (see Fig. 1). This configuration is of particular interest because of the application of the results to louver systems, cavity heat sinks, radiation shields and other engineering systems utilized in spacecraft. Furthermore, the results of analyses employing certain idealistic surface radiation properties are available for comparison and evaluation for selected values of the geometrical parameters.

In order to achieve a relatively simple formulation, retain a minimum number of important parameters, and to allow comparison to available results for diffuse and specularly reflecting gray surfaces, it is necessary to employ certain simplifying assumptions. First, surface heat exchange occurs only by thermal radiation and, except for the solar field, the contribution of external irradiation sources is negligible. Secondly, the intervening media neither emits, absorbs, nor scatters thermal radiation and furthermore possesses unit refractive index. Thirdly, the surfaces are opaque with uniform properties over their extent. The properties of both plates need not be identical. Finally, only steady state transport is considered and polarization effects on radiant energy exchange are neglected. Except for possibly the effects of polarization of which little is known, the foregoing simplifications do not impair the relevancy of the analysis to space applications.

The local radiant heat flux to a typical element of the lower plate, say dA_p, may be evaluated from the following expression:

$$q''(P) = \pi \int_0^\infty \int_{A_Q} I_\lambda(Q,\vec{r})\alpha_\lambda(P,\vec{r})K(P,Q)dA_Q d\lambda + \int_0^\infty G_\lambda(R,\vec{i})\alpha_\lambda(P,\vec{i})d\lambda$$
$$- \int_0^\infty \int_{2\pi} I_{\lambda,b}(P)\epsilon_\lambda(P,\vec{j})\cos\theta\, d\omega\, d_\lambda \qquad (2.1.1)$$

A similar expression may be written for the plate A_Q. The terms of equation (2.1.1) represent in the order in which they appear, absorbed irradiation from the plate A_Q, absorbed solar energy, and emitted energy of dA_p respectively. The contribution of solar irradiation appears in equation (2.1.1) only for elements directly illuminated by the solar field. The spectral distribution of the solar field is that proposed by Johnson [2]. K(P,Q) is a pure geometrical relation between the elements dA_P and dA_Q and has been determined in terms of the coordinate system affixed to the plate system shown in Fig. 1. Once the surface radiation properties are selected and the temperature distribution specified, the local radiant heat flux may be determined provided the spectral radiant intensity is available. The equilibrium temperature distribution of the surfaces can be calculated using an iterative procedure initiated with the results for local radiant heat flux obtained for the selected temperature variation [1].

The local spectral radiant intensity satisfies the integral equation

$$I_\lambda(P,\vec{s}) = I_{\lambda,b}(P)\epsilon_\lambda(P,\vec{s}) + \frac{G_\lambda(R,\vec{i})\rho_\lambda(P,\vec{i},\vec{s})}{\pi} +$$
$$\int_{A_Q} I_\lambda(Q,\vec{r})\rho_\lambda(P,\vec{r},\vec{s})K(P,Q)dA_Q \qquad (2.1.2)$$

where $\rho_\lambda(P,\vec{r},\vec{s})$ is the bi-directional (or biangular) reflectance. This property and its relation to other more commonly used radiation properties are discussed in Section 2.2. Equation (2.1.2) and a similiar equation for the spectral radiant intensity of the surface A_Q constitute a pair of simultaneous integral equations which require solution for each direction at every location on the surfaces.

It is of interest to demonstrate the correspondence of the foregoing analysis to the more idealistic theories for radiant energy exchange. Both theories in widespread use assume that emission is diffuse and treat reflection as either diffuse or specular. The possible dependence of the magnitude of reflectivity on the angle of incidence is not normally accounted for. Furthermore, spectral property dependencies are often disregarded (gray surfaces). These limiting cases are considered below.

The relevant integral equation for diffusely emitting and reflecting surfaces corresponding to equation (2.1.2) may be obtained by multiplying equation (2.1.2) by $\cos\theta d\omega$ and integrating over all possible directions $\vec{s}$ of hemispherical space above the element. Now a diffuse surface emits and reflects incident energy with equal intensity in all directions. Thus, a beam of incident energy regardless of direction of incidence is distributed uniformly over the hemispherical space above the surface after reflection. In this situation, the bi-directional reflectivity $\rho_\lambda(P,\vec{s}',\vec{s})$ is equal to the directional reflectivity $\rho_\lambda(P,\vec{s}')$. Furthermore, if the directional reflectivity is independent of the direction of incident radiation the hemispherical and directional reflectivities are equal. Accounting for the independence of I_λ on direction

and employing the relations between intensity, radiosity, and emissive power, the following result is obtained:

$$R_\lambda(P) = E_{\lambda,b}(P)\epsilon_{\lambda,H}(P) + \rho_{\lambda,H}(P) \int_{A_Q} R_\lambda(Q)K(P,Q)dA_Q + G_\lambda(R)\rho_{\lambda,H}(P) \tag{2.1.3}$$

If, in addition, the surfaces are gray, equation (2.1.3) may be integrated over the spectrum resulting in an analogous equation relating total energy quantities and properties. Numerical solutions to equation (2.1.3) have been reported [3] for gray plates of identical uniform properties and temperatures in the absence of external sources of irradiation.

For specularly reflecting surfaces, the angles of incidence and reflection as well as the solid angles of transfer for incident and reflected radiation are equal. Designating by $\vec{s}_{sp}$ the direction of the incident ray which is specularly reflected into the $\vec{s}$ direction, the integral term in equation (2.1.2) reduces to simply $\rho_\lambda(P,\vec{s}_{sp})I_\lambda(Q,\vec{s}_{sp})$.

Thus,

$$I_\lambda(P,\vec{s}) = I_{\lambda,b}(P)\epsilon_\lambda(P,\vec{s}) + \rho_\lambda(P,\vec{s}_{sp})I_\lambda(Q,\vec{s}_{sp}) + \rho_\lambda(P,\vec{s}_{sp})G_\lambda(P,\vec{s}_{sp}) \tag{2.1.4}$$

Equation (2.1.4) has in principle been solved for gray surfaces in the absence of external irradiation sources [4].

Numerical solution of equation (2.1.2) is required. Two numerical methods are under consideration, namely iterative and Monte Carlo techniques. In order to assess the relative merit of these techniques and investigate any numerical difficulties which may arise, present efforts are directed at the

solution of a simpler geometrical system. By restricting consideration to two thin strips as shown in Fig. 3, all elements of the analysis are retained except for the influence of the plate width.

2.2 Radiation Properties for Rough Surfaces of Conducting Materials

The definition for bi-directional reflectance chosen in this work is that of Polyak [5], and is defined such that (Fig.2)

$$\rho_\lambda(P,\vec{r},\vec{s})\ \frac{\cos\theta d\omega}{\pi} \tag{2.2.1}$$

is the fraction of the energy within the spectrum λ to $\lambda + d\lambda$ incident at P from the $\vec{r}$ direction which is reflected into the solid angle $d\omega$ about the $\vec{s}$ direction. For definitions used by other investigators, see Table I. The corresponding monochromatic directional, total directional, monochromatic hemispherical, and total hemispherical reflectances can be obtained by appropriate integration over the incident and reflected solid angles and over the wave length spectrum.

It was proposed [1] that the bi-directional reflectance model derived by Davies [6] be used in this investigation. This model describes scattering by a perfectly conducting surface with a Gaussian distribution of surface height. Beckmann [7] points out that Davies used a form of the Helmholtz integral which is applicable only for roughness elements with small slopes.

Birkebak [8,9] used Davies' equations to correlate bi-directional reflectance data for monochromatic irradiation of nickel and aluminum samples of various roughnesses. These measurements were limited to radiation incident at 10° to the surface normal. It appears that Birkebak had overlooked the corrections to Davies' equation for the case when $\sigma/\lambda >> 1$. The corrected

form [11] of Davies' result in terms of the bi-directional reflectance of equation (2.2.1) is

$$\rho(\theta,\varphi,\psi,\pi)\cos\theta = \frac{1}{16\cos\psi}\left(\frac{a}{\sigma}\right)^2 \exp\left\{-\left(\frac{a}{2\sigma}\right)^2\left[\frac{(\sin\theta\cos\varphi-\sin\psi)^2+\sin^2\theta\sin^2\varphi}{(\cos\theta+\cos\psi)^2}\right]\right\} \tag{2.2.2}$$

Preliminary calculations to determine the affect of this error on the substantiation of Davies' model by Birkebak indicate that the model may still be applicable. It appears, however, that the range of the parameter a/σ lies between 30 and 60 rather than 10 and 20 as reported [8].

Since the surface in Davies' work was assumed perfectly conducting the model should predict a directional reflectance of unity independent of the parameters σ/λ, a/σ, and angle of incidence. Beckmann [7] gives a qualitative discussion of the validity of accounting for finite conductivity by simply introducing the multiplicative factor $\rho_o(\psi,\pi)$, the Fresnel reflectance. In general it is not possible to account for finite conductivity in this manner.

Beckmann [7] develops a model that appears superior to that of Davies'. This model is only slightly more complicated and is less restrictive in terms of the local roughness slopes.

Further investigation of the correlation of Birkebak's data with Davies' model is necessary in view of the incorrect equation used in[3,9]. The accuracy of Davies' model at large angles of incidence, and the present method of accounting for finite conductivity also require study.

2.3 Adjoint Plate System Analysis

The adjoint plate system (see Fig. 4), which is a special case of the geometry considered in Section 2.1, has received considerable attention. Analyses for diffuse emission and both limiting cases of reflection (diffuse and specular) have been reported in the absence of external radiation fields when the plates are gray and at identical uniform temperatures [3,4].

Theory and experiment indicate that the emissivity and reflectivity of optically smooth surfaces depend upon the direction of emission and direction of incidence respectively. An analysis was completed accounting for directional dependence of radiation properties on radiant heat transfer. In view of the results already available, the adjoint plate system was selected for study. The plates are assumed gray specular reflectors with emission and reflection properties as characterized by the electromagnetic theory for smooth electrical conductors and non-conductors. Each plate is at a uniform temperature and has uniform properties over its extent, although the temperature and properties of the plates need not be identical. The dimensionless local radiant heat loss of the lower plate, $q''_L/\epsilon_{H,L}E_{b,L}$ is determined by the following expression:

$$\frac{q''_L(\xi)}{\epsilon_{H,L}E_{b,L}} = 1 - \left(\frac{1}{\epsilon_{H,L}}\right)\left(\frac{E_{b,U}}{E_{b,L}}\right)\left(2\int_0^1\int_0^\infty G_\gamma d\eta' d\xi'\right) \qquad (2.3.1)$$

The dimensionless local radiant heat loss depends upon the included angle between the plates (γ), the ratio of plate emissive powers ($E_{b,U}/E_{b,L}$), and the selected variation of emissivity and reflectivity with direction of emission and incidence. In order to compare the results with those already

available, emissivity and reflectivity distributions must be chosen which result in hemispherical emissivities of 0.1, 0.5, and 0.9. The property variations chosen are discussed in Section 2.4.

As noted previously, the analysis is completed. A computer code has been written and verified. Calculations are in progress for opening angles of 45°, 60°, 90°, and 135° for hemispherical emissivity values of 0.1, 0.5, and 0.9 with plate emissive power ratios of 0.5, 1.0, and 2.0. Preliminary results indicate that local radiant heat losses may be significantly affected but total heat loss is predicted reasonably well by the simplified theory for diffusely emitting-specularly reflecting surfaces.

2.4 Radiation Properties of Optically Smooth Plane Surfaces

The results of the diffusely emitting-specularly reflecting analysis of the adjoint plate system [4] are based on hemispherical emittance values of 0.9, 0.5, and 0.1. In order to use these results to assess the importance of directional property variations on the heat transfer, it is necessary to employ directional property distributions which are consistent with the available results. Electromagnetic theory provides directional reflectivity and emissivity relationships for optically smooth non-conductors and conductors of electricity in terms of the refractive and/or absorption index of the material. Since hemispherical properties are not generally available in terms of these material parameters, limited calculations were performed to ascertain the values of these parameters which result in the appropriate value of the hemispherical property. For infra-red radiation non-conductors are characterized by large and conductors by small values of emittance. Therefore, directional

distributions corresponding to dielectrics are employed for surfaces with hemispherical emittances of 0.9 and 0.5 and a directional distribution corresponding to a metal for hemispherical emittance of 0.1. A brief summary of the results are presented below.

Non-conductors According to the electromagnetic theory, the reflectivity of smooth plane surfaces for unpolarized radiation is given by the relation

$$\rho(\theta) = \frac{1}{2}\left\{\left[\frac{\cos\theta - \sqrt{n^2-\sin^2\theta}}{\cos\theta + \sqrt{n^2-\sin^2\theta}}\right]^2 + \left[\frac{n^2\cos\theta - \sqrt{n^2-\sin^2\theta}}{n^2\cos\theta + \sqrt{n^2-\sin^2\theta}}\right]^2\right\} \qquad (2.4.1)$$

where θ is the angle of incidence and n the refractive index of the dielectric. The directional emissivity $\epsilon(\theta)$ may be taken as $1-\rho(\theta)$.

The hemispherical emissivity is given in terms of the directional emissivity by

$$\epsilon_H = \int_0^{\pi/2} \epsilon(\theta)\, \sin 2\theta\, d\theta \qquad (2.4.2)$$

Account has been taken of the azimuthal symmetry of $\epsilon(\theta)$ in equation (2.4.2).

Results obtained by digital computation for hemispherical, normal, and the ratio of hemispherical to normal emissivity are presented in Table II for selected values of the refractive index. Slight discrepancies exist between these results and those of [13] which were obtained by graphical integration. Figures 5 and 6 display these results in graphical form. The values of the refractive index which result in hemispherical emissivities of 0.9 and 0.5 are 1.5565 and 6.1038, respectively. The directional emissivity distributions corresponding to these values of the refractive index are shown in Fig. 7.

Conductors An approximate relation for the directional emissivity of good electrical conductors for wave length outside the visible spectrum [13] is

$$\epsilon(\theta) = 1 - \frac{1}{2}\left\{\frac{(n-\cos\theta)^2 + n^2}{(n+\cos\theta)^2 + n^2} + \frac{(n - 1/\cos\theta)^2 + n^2}{(n + 1/\cos\theta)^2 + n^2}\right\} \qquad (2.4.2)$$

Assuming independence of n on wave length, the value of the real part of the refractive index of metals (n) for which the hemispherical emissivity is 0.1 is 23.452 . The directional emissivity for this value of refractive index is shown in Fig. 7.

3. PROPOSED FUTURE RESEARCH

The influence of non-diffuse non-gray surface radiating properties on radiant transfer will be investigated further. Computer codes for the strip geometry will be developed and results obtained for non-gray non-diffuse surfaces. Further investigation of the bi-directional reflectance models is required to ascertain the degree to which these models are substantiated by available data. Details of the analysis, results, and conclusions for the system considered in Section 2.3 will be prepared for publication upon completion. Copies of this manuscript will be submitted to NASA. Additional analyses including the effects of directional property variations for non-gray surfaces will also be initiated.

4. REFERENCES

1. R. G. Hering, "Theoretical Study of Radiant Heat Exchange for Non-Gray, Non-Diffuse Surfaces in a Space Environment," Proposal Resulting in NASA Research Grant No. NGR-14-005-036.

2. F. S. Johnson, "The Solar Constant," Jour. Meteorol., 11, No. 6, 1954, pp. 431-439.

3. E. M. Sparrow, J. L. Gregg, J. U. Szel, and P. Manos, "Analysis, Results, and Interpretation for Radiation between Some Simply Arranged, Gray Surfaces," Jour. of Heat Transfer, Trans. ASME, Series C., Vol. 83, 1961.

4. E. R. G. Eckert & E. M. Sparrow, "Radiative Heat Exchange between Surfaces with Specular Reflection," Int. Journ. Heat, Mass Transfer, 3, pp. 42-54, 1961.

5. G. L. Polyak, "Radiative Transfer between Surfaces of Arbitrary Spatial Distribution of Reflection," Translation TT-9, Radiative Transfer Project, Purdue University, Lafayette, Indiana.

6. H. Davies, "The Reflection of Electromagnetic Waves from a Rough Surface Proc. I.E.E., Pt. III, 101, pp. 209-214, 1954.

7. P. Beckmann, A. Spizzichino, "The Scattering of Electromagnetic Waves from Rough Surfaces," The Macmillan Company, New York, 1963.

8. R. C. Birkebak, "Monochromatic Directional Distribution of Reflected Thermal Radiation from Roughened Surfaces," Ph.D. Dissertation, Mechanical Engineering Department, University of Minnesota, Minneapolis, Minnesota, September 1962.

9. R. C. Birkebak, E. R. G. Eckert, "Effects of Roughness of Metal Surfaces on Angular Distribution of Monochromatic Reflected Radiation," Jour. of Heat Transfer, February 1965, pp. 85-94.

10. D. K. Edwards & J. T. Bevans, "Radiation Exchange in an Enclosure with Directional Wall Properties," ASME Paper, 64 WA/HT-52.

11. L. M. Spetner, Discussion on [6]. Proc. I.E.E., Pt. III, 102, 148, 1955

12. K. E. Torrance & E. M. Sparrow, "Discussion at end of [9].

13. M. Jakob, "Heat Transfer," John Wiley and Sons, 1949.

5. FIGURES AND TABLES

Fig. 1 Geometry of Analysis

Fig. 2 Angles of Incidence and Reflection

Fig. 3 Thin Strips Geometry

Fig. 4 Adjoint Plate System

Fig. 5 Theoretical Values of Hemispherical and Normal Emissivity of Dielectrics

Fig. 6 Theoretical Values for the Ratio of Hemispherical to Normal Emissivity of Dielectrics

Fig. 7 Theoretical Directional Emissivity Distributions

Table I Bi-directional Reflectance Definitions

Table II Emissivity of Dielectrics for Selected Values of the Refractive Index

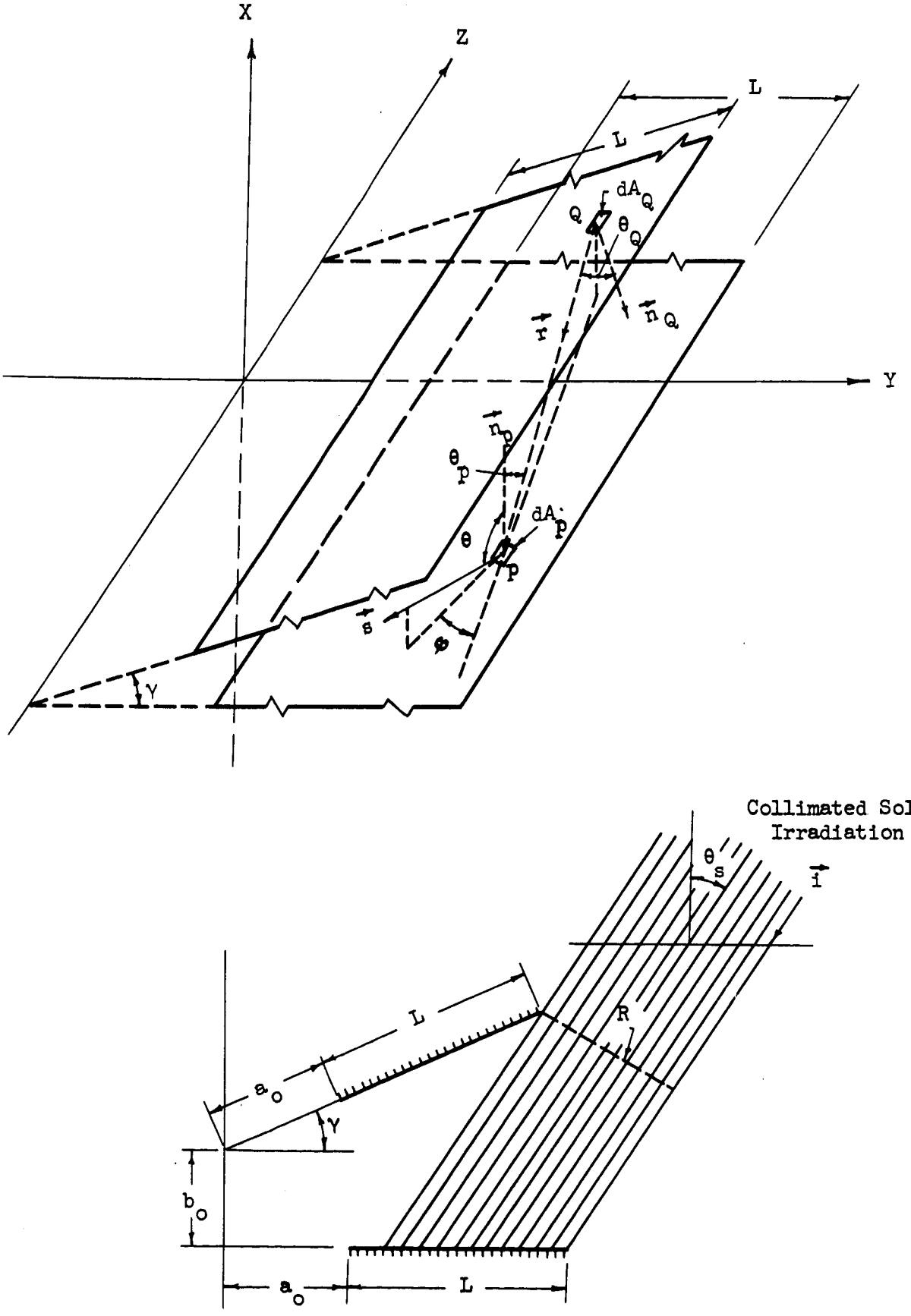

Figure 1

Geometry of Analysis

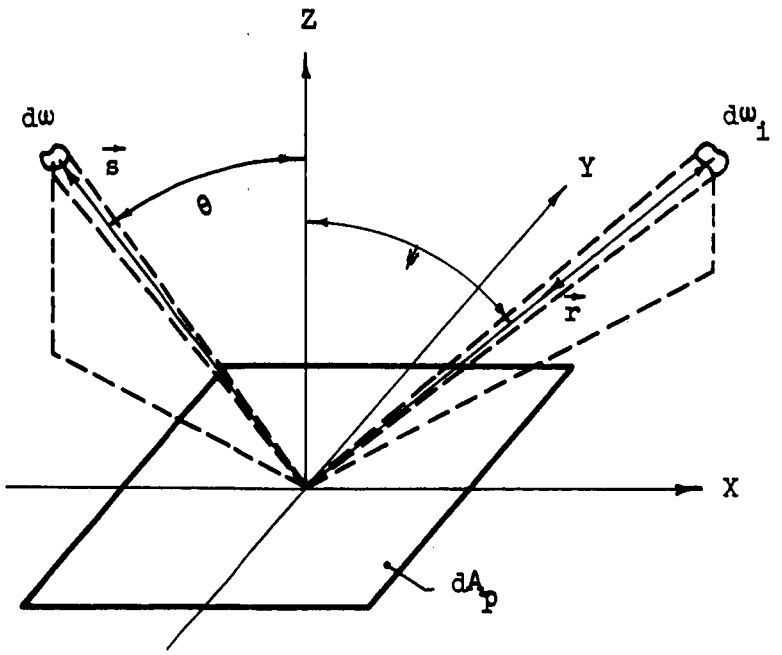

Figure 2
Angles of Incidence and Reflection

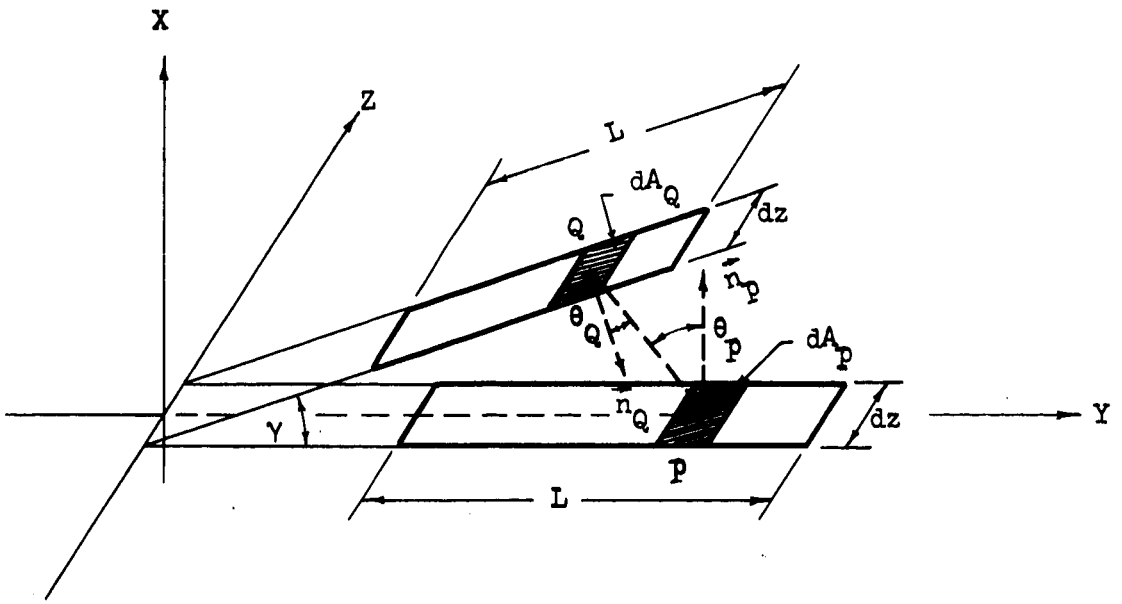

Figure 3
Thin Strips Geometry

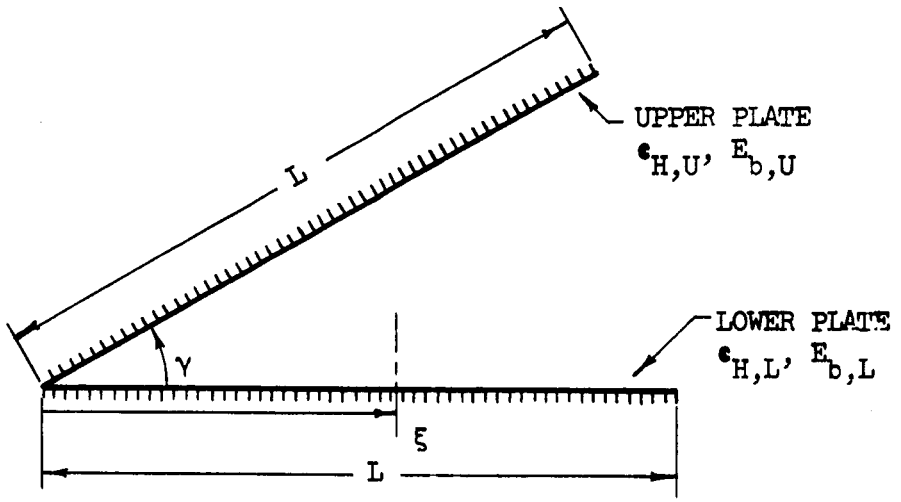

Figure 4

Adjoint Plate System

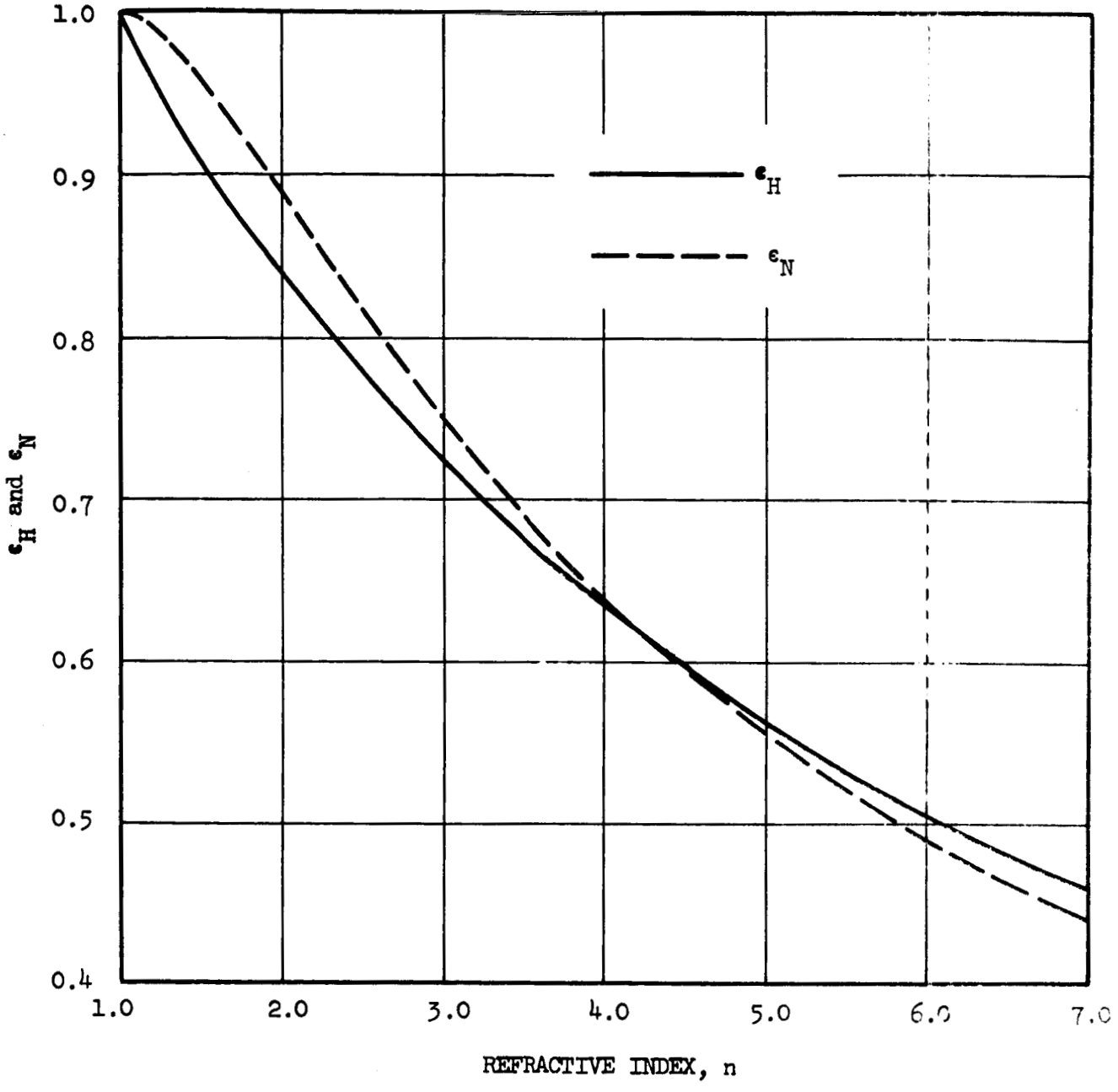

Figure 5

Theoretical Values of Hemispherical and Normal Emissivity of Dielectrics

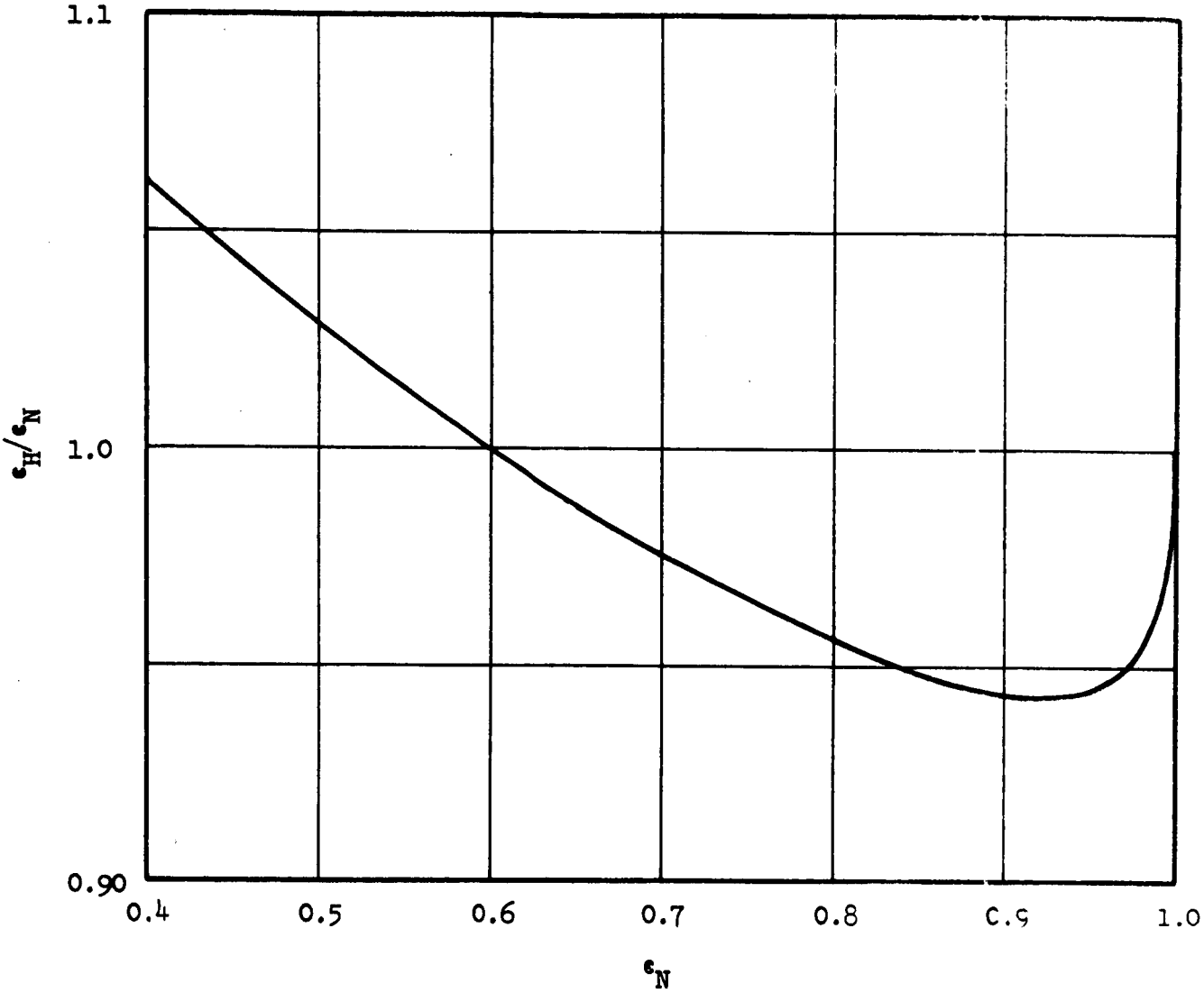

Figure 6

Theoretical Values for the Ratio of Hemispherical to Normal Emissivity of Dielectrics

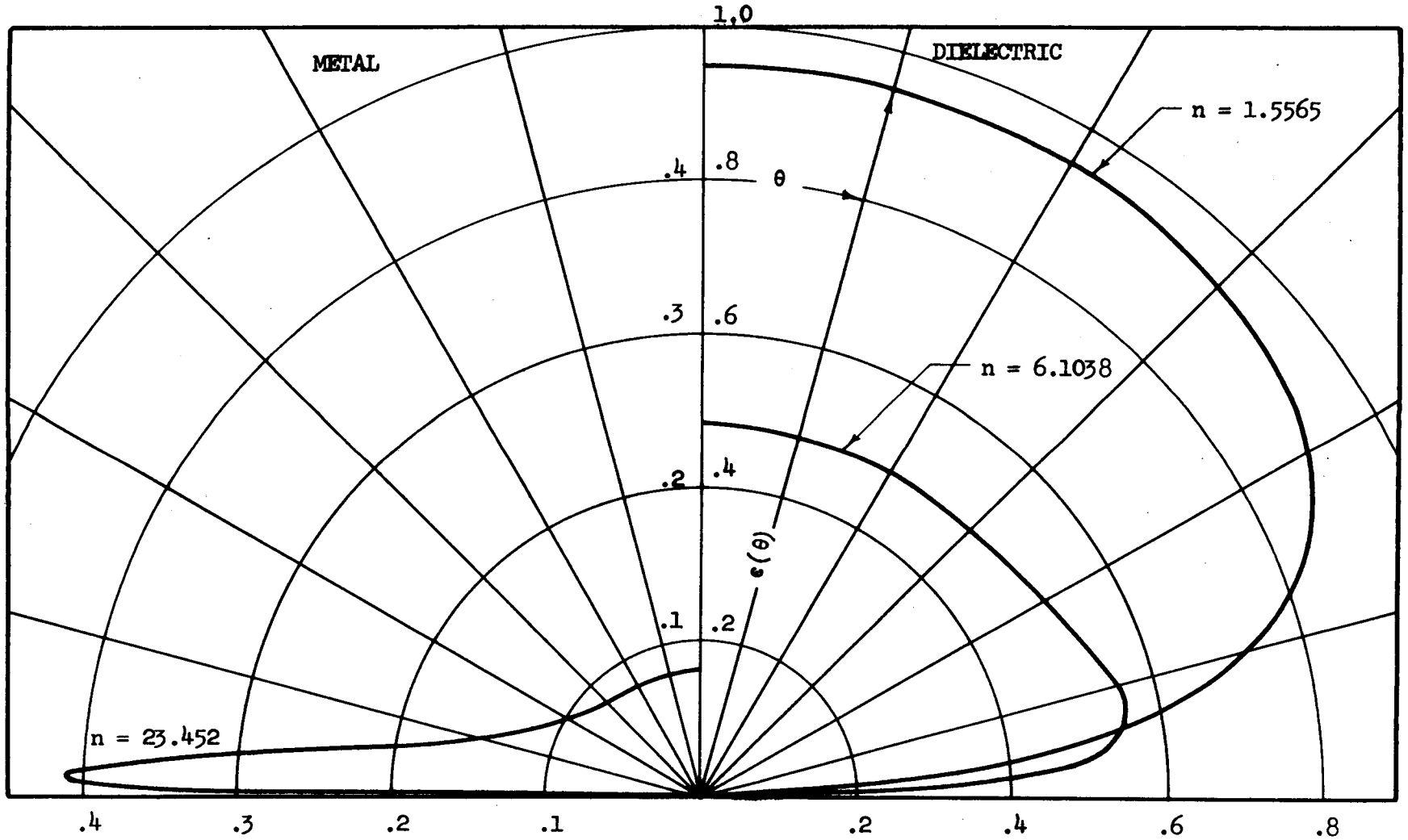

Figure 7

Theoretical Directional Emissivity Distributions

TABLE I

BI-DIRECTIONAL REFLECTANCE DEFINITIONS

Investigator	$\rho_\lambda(P,\vec{t},\vec{s})$
Polyak [5]	$\dfrac{\pi\, I_{r,\lambda}(P,\vec{s})}{I_{i,\lambda}(P,\vec{t})\cos\Psi\, d\omega_i}$
Birkebak [8] Edwards & Bevans [10]	$\dfrac{I_{r,\lambda}(P,\vec{s})}{I_{i,\lambda}(P,\vec{t})}$
Sparrow & Torrance [12]	$\dfrac{I_{r,\lambda}(P,\vec{s})}{I_{i,\lambda}(P,\vec{t})\cos\Psi\, d\omega_i}$

*Above definitions are in terms of nomenclature used in this publication.

TABLE II

EMISSIVITY OF DIELECTRICS FOR SELECTED VALUES OF THE REFRACTIVE INDEX

n	ϵ_H	ϵ_N	ϵ_H/ϵ_N
1.000	1.000	1.000	1.000
1.410	0.9217 (0.91)	0.9711	0.9491
1.500	0.9082	0.9600	0.9461
1.5565	0.9000	0.9526	0.9448
1.576	0.8972	0.9500	0.9444 (.935)
1.9250	0.8492	0.9000	0.9436 (.935)
2.000	0.8394	0.8889	0.9443
2.2642	0.8061	0.8500	0.9484 (0.94)
2.500	0.7781	0.8163	0.9532
2.618	0.7647	0.8000	0.9559 (0.95)
3.000	0.7238	0.7500	0.9651 (0.96)
3.4221	0.6829	0.7000	0.9756 (0.97)
3.8973	0.6417	0.6500	0.9872 (0.98)
4.000	0.6334	0.6400	0.9896
4.4415	0.5999	0.6000	0.9999 (0.95)
5.000	0.5622	0.5556	1.0120
5.0757	0.5575	0.5500	1.0136 (1.005)
5.8284	0.5142	0.5000	1.0284 (1.025)
6.000	0.5053	0.4898	1.0316
6.1038	0.5000	0.4838	1.0334
7.000	0.4588	0.4375	1.0486

*Values in parentheses are from [13].